general, a groupoid is a small category in which each map is an isomorphism; equivalently, it is “a [small] category in which each edge (morphism) is invertible” (Higgins, 2005, p. 5). In this context, the relation between group and groupoid is apparent. The symmetry group of an object is its automorphism group as defined on p. 262. A group is actually the specific case of a groupoid having just one object. In other words, “a group is a one object groupoid and a groupoid is a many object group” (Brown & Porter, 2006, p. 265). Note that some or all of the objects in a groupoid may be non-terminal objects (so that, as groupoids, the examples in Fig. 9.7 are atypically simple). Note also that Fig. 9.7 is not a “diagram” like those seen earlier: in it, the arrows do not denote isomorphisms, i.e., maps, but rather the relationship of isomorphism between the objects they join—there must be at least one map-isomorphism in each direction, but there may be many. Also, Fig. 9.7 includes no visible indication of the possibly very extensive group of automorphisms at each object. We are operating at an even higher level of abstraction than before.

A general groupoid operation is a composition of maps between different objects and automorphisms of those objects, e.g.,

\[ \alpha \circ A \rightarrow A \rightarrow f \rightarrow B \rightarrow g \rightarrow B \rightarrow \gamma \rightarrow C \rightarrow C \]

where \( \alpha, \beta \) and \( \gamma \) are automorphisms of \( A, B, \) and \( C \) respectively, and \( f: A \rightarrow B, g: B \rightarrow C \) are isomorphisms. In a complex system, sequences of groupoid operations result in ‘symmetric structure’; i.e., there is a sequence of morphisms that ends back at the object where it originated. In a complex system it may be difficult to identify such

Figure 9.7: Groupoids with three, four, and six objects, respectively.

---

9A category is called “small” in case the collection of all its maps is a set (i.e., not a so-called ‘proper class’ like the collection of all sets, which is not itself a set).
a path, but the notions of a groupoid and of a symmetric structure associated with it allow us to identify a pattern better than we could based on the concept of group *per se*. As Stewart (2004, p. 603) argues, “Groupoids have, on the whole, been somewhat removed from the mathematical mainstream [...]. However, they are the ideal tool for describing symmetries that apply only to *parts of the systems*” (emphasis in the original).

**Groupoids as ‘boundary objects’**

A groupoid like those illustrated in Fig. 9.7 involves (1) product/co-product, and (2) at least three terminal/initial objects. I would like to suggest that such a structure be described as a *boundary object*, and the objects constituting it as *boundary sub-objects*. I would also like to suggest that, as islands of symmetry/reversibility, these boundary objects constitute and regulate the structure of the system from within. These groupoids are boundary objects because they are “islands of reversibility” that ensure the identity of the system from within. Following this suggestion, a structure/pattern cannot be considered in terms of a simple object that preserves its symmetry under transformations; rather, it is an abstract relational system that emerges from local symmetries between several objects. Moreover, the identity of the pattern is determined through the boundary objects and the morphisms that connect them. More specifically, the pattern is constituted by the mapping of this category across time.

To present a formal characterization of structure, we need to introduce one last concept from category theory, that of a *functor*, *i.e.*, a mapping from a category $C_1$ to a category $C_2$ that assigns objects to objects and maps to maps in such a way that (1) composition is preserved and (2) identity maps of $C_1$ map to identity maps of $C_2$.

**A minimal definition of structure using category theory**

As Piaget realized, cognitive and biological systems are dynamic systems that evolve over time. Therefore, structure simply does not exist in the static sense and would be better described in dynamic terms. Following Ehresmann and Vanbremeersch (2007) we can model such a dynamic system with the concept of a *state-category*. A state-category is actually a directed graph interpreted as a category. The change in the system from $t$ to $t'$ (for $t' > t$) can be